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I. THE FAMILY TREE

Lanchester descriptions of ground combat have pervaded the area of war gaming for the past twenty years. This is due both to the appealing tractability attained by describing combat as a continuous process in which attrition is a simple function of force strengths, and to the current lack of acceptable alternative methods of describing the combat effectiveness of units possessing equipment not yet built, operating in combat environments not yet experienced.

Paralleling the increased and accelerating availability of computing power, inquiry has been directed at force composition, weapon mixes, logistic support levels, and equipment development programs. The computerized ground combat models developed to explore these issues have either been extensions of the Lanchester formulation, or detailed simulations, in which attrition, movement and tactical decisions are determined separately for each element. The former have been unsatisfying due to their gross and incomplete treatment of combat, and the latter have generally been operationally and computationally unwieldy.

In this paper we will briefly trace the development of a family of differential combat models, describe the current model in this family, and based on the insight gained in experimenting with this model, state several directions in which improvement appears necessary and possible.

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The progenitor of the present line was MINIWAR, an analog computer model constructed in 1962 for descriptive and instructional purposes by David Howes of the U.S. Army Strategy and Tactics Analysis Group (STAG). MINIWAR was a simple area fire description of the attrition suffered by two units

$$\frac{dR}{dt} = k_1 RB \tag{1}$$

$$\frac{dB}{dt} = k_2 RB \tag{2}$$

where R and B are scalar measures of the effective strengths of the two units, and k_1 and k_2 are factors determining the rate of attrition. During 1963 this simple area fire formulation of combat was transferred to a digital computer and was extended to consider the interaction of the firepower and movement of several units. Units were individually assigned unopposed speeds appropriate for the terrain, and were assumed to move at these speeds less degradation due to hostile fire. For a single unit the movement rate is

$$\frac{dD_R}{dt} = S_R - f(R,B,S_R) \tag{3}$$

where D_R is the instantaneous movement rate of R, S_R is the unopposed movement rate of R, and $f(R,B,S_R)$ is the degradation of S_R due to incoming hostile fire. During the early part of 1964 the model gained in complexity. Unit location, weapon mix, mission, and targeting doctrine were added. Units were no longer defined by scalar firepower scores but by the number of personnel and weapons possessed. Weapons were described by their range and lethality characteristics, and unit behavior became dependent on the mission and the situation. Units were composed of weapons of different types, but the attrition equations remained formulated in terms of overall effective strength.

Therefore a set of weapon relative effectiveness matrices were developed which were used to reduce the weapon descriptions of units to single numbers. This model, named Tartarus I, was programmed for the IBM 7090 and was used as the computing portion of an on-line, man-machine game. This mode of operation permitted rapid computer assessment of mobility and firepower interactions while allowing manual intervention to reinforce units, change missions or locations, or introduce new tactical data.

During the remainder of 1964 the process of weapon attrition, the movement logic of units, and the targeting process were refined. When the model was extended to allow air strikes against any of the weapon classes possessed by ground units, the name changed to Tartarus II.

At that point development of the model reached a branch point. The outcome of ground combat depends on the lethality characteristics of weapons possessed, the mobility characteristics of weapons and equipment possessed, and on the command, control, and communication structure of the organizations involved. The lethality potential of weapons is dependent on the ability of a supply system to provide ammunition, and in 1965 The RAND Corporation created Tarlog, which extended the Tartarus II model to explicitly consider the effect of ammunition supply rates on the combat performance of ground combat organizations. At the same time STAG continued the development of Tartarus (now in version IV) to include the effect of terrain on movement, and the effect of intelligence and communications on combat performance.

Thus while much of the behavioral logic and secondary structure has been altered, Tarlog retains the basic attrition and mobility characteristics of Tartarus II, but causes weapon performance to depend directly on the availability of ammunition.

We have found Tarlog useful for comparing alternative forces, but not appropriate for making absolute predictive statements about combat outcomes. This is due to the current lack of a convincing analytic description of the manner in which various weapon types interact, and the effect of hostile fire in retarding the movement of a combat unit. The first steps in developing a model with predictive characteristics

may be a separate attrition equation for each weapon class, and a separate equation relating the movement of every mobile weapon and equipment class to the hostile incoming fire. The proportionality factors and coefficients for such systems of equations could be derived from detailed simulation models such as Carmonette or Fast-Val, thus combining the rapid assessment capability of a differential model with the greater precision of an elemental simulation.

II. ELEMENTS AND DATA REQUIREMENTS

The basic elements of the Tarlog model are the combat units. Tarlog operates on the combat units, altering their composition and location as the battle proceeds. Units are initially described by their name, number of personnel, and number of each of sixty weapon types. Units are assigned initial locations, intermediate and final objectives, and widths. They are then conceived of by the model as lines perpendicular to the path joining the current location to the first objective with half of the unit width on either side of the path. Units are assigned one of five missions which determine their behavior. These are attack, prepared defense, hasty defense, counter-attack, and delay to an objective. Each unit is assigned an initial basic load of ammunition, an ammunition supply rate in tons per day, and a resupply cycle time. Finally units each have designated levels of personnel loss at which they will change missions.

Sixty weapon types can be defined for use in Tarlog. Weapons are described by effective maximum and minimum ranges, average lethal area per round, average round weight, and the rate of fire the weapon and crew are physically capable of sustaining. Weapons are assigned to one of ten weapon classes. The model allows the definition and use of twelve types of air weapons. These must be defined by the quantity of lethal area contained in a single aircraft load of the weapon type.

Four proportionality constants are required as input data. These control the rate of attrition caused by incoming fire, suppression of firepower output due to incoming fire, reduction in unit firepower

output due to unit movement, and reduction in unit speed (or ability to hold) due to incoming fire.

The model also uses five matrices of numbers, E_{ijm}, which describe the relative effectiveness of each ground weapon class against each other ground weapon class for each of the five missions of the firing unit, and two matrices of numbers, E_{Ajm}, which perform the same function for air weapons.

The Tarlog model determines the number of personnel casualties from the number of weapons lost. Therefore a required data input is the number of personnel casualties when a single weapon in each of the ten weapon classes is lost. The model also uses personnel casualties to degrade unit combat effectiveness. This is done by introducing as input a profile of unit combat effectiveness as a function of personnel casualties.

III. MODEL STRUCTURE AND OPERATING CHARACTERISTICS

We will first present a general description of the Tarlog model, and then discuss the structure and logic of the various parts in more detail.

In the Tarlog model attrition and movement are described by a set of differential equations. We have experimented with two formulations of the attrition process. The first is appropriate for describing the situation in which units fire uniformly into an area occupied by their targets. In the simplest formulation of this case the equilibrium solution is the Lanchester "linear" law. The second formulation assumes that the defender does not fire into an area, but can bring fire to bear on specific target weapons, and shift fire appropriately when targets are destroyed. In both formulations, the factors in the attrition equation are dependent on the immediate situation. The number of units firing, weapon mix, weapon ranges, movement rate, and ammunition supply are all considered in determining the attrition suffered by each target unit.

Movement degradation in Tarlog is essentially proportional to the square root of the effective firepower ratio. The precise formulation

of the instantaneous movement rate of a unit, N, is

$$\frac{dD_N}{dt} = S_{BN}\left[1 - k_{Nm}\sqrt{\frac{U_N(t) + A_N(t)}{L_N(t) \cdot \mu_N(t)}}\right] \tag{4}$$

where S_{BN} = the unopposed basic speed of N,

k_{Nm} = the movement degradation proportionality factor dependent on N's mission, m,

$U_N(t)$ = the total effective ground fire delivered on N,

$A_N(t)$ = the total effective fire delivered by air weapons on N,

$L_N(t)$ = the total firepower potential of N,

$\mu_N(t)$ = a combat effectiveness multiplier $(0 \le \mu \le 1)$ dependent on the personnel casualties suffered by N.

Equation (4) produces reasonable movement behavior for a wide spectrum of firepower and speed values. hen the model is used as a tool for comparing organizational behavior and not predicting battle outcomes, the movement relationship is credible and satisfactor.

In the Tarlog computer program a land battle is carried out in a series of combat intervals of equal length, during which the combat units move toward assigned objectives and deliver fire on opponents. Up to fifty units of arbitrary composition can be played in the model. Each unit is conceived of as a line with half its width on either side of the path joining the unit's location and its current objective. Units are further described by their personnel, weapons, and mission. Air strikes can be directed against ground units, and can utilize one of twelve types of air to ground weapons. As units suffer attrition during the combat intervals, their ability to hold ground or advance changes. If a sufficient number of personnel are lost by a unit, the mission may change or the unit may break, stopping the game.

The program is organized into a preprocessor section and a combat simulation section. The preprocessor contains the subroutines required

to read the various types of input data and serves as a comprehensive editing and diagnostic program. Input data irregularities discovered by the preprocessor do not terminate input editing, but do prevent entry into the combat simulation section.

In the combat simulation section processing is done at the start of each combat interval, during the combat interval, and after each combat interval. At the start of the combat interval unit firepower potential is computed as a function of the available ammunition, and each unit selects targets on the basis of mission, range, and weapons possessed. During the combat interval unit movement rates and attrition are appropriately modified by the effect of air and ground action. If critical events occur such as units reaching objectives or personnel strengths falling below preset values, the routine will either stop or cause changes in unit missions or objectives. Following the combat interval the total firepower loss suffered by each unit is apportioned into losses in each of the unit's ten weapon classes on the basis of the effectiveness of the various weapons involved, and the time interval during which they were able to fire. At this point the combat effectiveness profile is entered with current personnel casualties to determine the combat effectiveness factor for each unit. Requested status reports are then printed out and the computations for the next combat interval are begun. The routine terminates either when the end time is reached, when a unit falls below a final personnel level, or when a final objective is attained. On termination, the routine will provide a restart deck suitable for continuing play from the termination time.

The entire program is written in the FORTRAN IV language and is designed to operate with the IBM 7040/7044 Operating System. Execution time, including input and output, depends on the number of units involved, the length of the combat interval, and the length of the iteration interval used in the solution of the differential equations. We have used the estimate that one second of execution is required per unit-hour of combat simulated when the combat interval is two hours and the iteration interval is two minutes.

COMBAT UNIT DESCRIPTIONS

Combat units are initially described by their names, the number of personnel, and the number of each of sixty different weapon types. We aggregate each unit's weapons into ten weapon classes according to a designated type to class assignment schedule. Since each weapon type has been defined by lethal area per round and potential sustained rate of fire, we compute a weapon class firepower potential for each weapon class. We next find the total number of weapons in each weapon class, term these "equivalent" weapons, and compute an equivalent lethal area per round and equivalent sustained rate of fire for the weapons in each class in each unit. We also compute equivalent maximum and minimum ranges and round weights for each of the ten weapon classes in each unit. After this initial aggregation is accomplished Tarlog operates either at the weapon class firepower level or at the level of unit total firepower potential.

An additional set of data is entered for each unit which determines its combat behavior. This data includes the unit mission, unopposed speed, location, objective(s), width, priority targets, fire restrictions, time at which the unit is free to move, mission change levels, and ammunition supply data. The model is structured for prepared defense, attack, counter-attack, hasty defense, and delay to objective missions. The mission designation determines the effectiveness of firepower output, vulnerability, and the movement behavior of the unit. Unit locations and up to six geographical objectives are described in arbitrary coordinates which however must be consistent with the measurement units of the speed. The first five objectives are generally used as points on a broken line advance path, while the sixth is always the location of a point to which the unit may retreat if its mission changes to delay. Each unit is always considered to have half its width on either side of the path connecting its current location and current objective, with its front perpendicular to this path. The shortest distance between two units is the shortest distance between their two lines. Normally each unit seeks four targets at the start of each combat interval. However, two opposing units can be assigned to a given unit as priority targets. These targets will be retained throughout the

entire run, but if the priority targets are not within range of the firing unit, only two transient targets can be selected at the start of the combat interval. A "zone fire" restriction can be imposed on any unit and will prevent that unit from firing at any unit not immediately forward of the firing unit's front, and a unit may be assigned an earliest time at which it is free to move. Each unit may be assigned a basic ammunition load by tons per weapon class, an ammunition resupply rate in tons per day, and a resupply cycle time. The effect of ammunition will be described in the next paragraph. Finally, a series of air strikes may be scheduled against a unit or against one or two specified weapon classes within the unit. If the directed weapon classes are not present, or if the target unit is not present when the strikes occur no other unit or weapon class within the unit will be affected.

AMMUNITION RESUPPLY, ALLOCATION, AND RATE OF FIRE COMPUTATION

Rates of fire are based on ammunition available. Each unit receives stocks of ammunition which first fill weapon classes to their basic loads, and only then does the unit assign extra ammunition to high usage weapon classes. Based on the tonnage available to each weapon class and a predesignated safety horizon of ammunition use, we compute a rate of fire which will conserve the ammunition for the prescribed period.

Each unit has a basic load specified in tons of ammunition per class, a resupply rate in tons per day, and a resupply interval in hours. The appropriate fraction of the resupply tonnage is delivered to the unit at the completion of each resupply interval. We assume that while the supply system is aware of the time of resupply, the unit is not, and therefore it cautiously computes that rate of fire which will use up existing stocks of ammunition over the predesignated safety horizon -- say fifteen hours. Since the rate of fire computation is done at the start of each combat interval, almost certainly less than fifteen hours, if no supply arrives the firing rate will continually decrease, but the ammunition on hand will never be totally expended.

At the start of the first combat interval the model simply computes the rate of fire which if continued for fifteen hours would totally deplete the basic load in each class. Using this rate of fire the firepower potential in the weapon class is computed, and then summed to form the unit total firepower potential.

During the combat interval some weapon classes may not be within range of the unit's targets, and consequently will not fire, and some basic load requirements will decrease due to destruction of weapons. Therefore after the first period we use the following ammunition allocation procedure. If ammunition resupply is due at the end of a combat interval we consider the arriving ammunition undetermined by class, and assign the tonnage to a "reserve" within the unit. We then adjust the amount of ammunition remaining in each class to account for the ammunition expended during the preceding combat interval, and reduce the basic load required by each weapon class to account for the number of weapons in the class which have been destroyed during the combat interval. Ammunition in the unit reserve is then used to fill all weapon classes back toward their basic loads. If a weapon class is already at its basic load, we assign it nothing, and if there is insufficient ammunition to fill all weapon classes, we allocate ammunition to each class in proportion to its tonnage deficit. Priority is thus given to refilling basic loads and we assume that the supply system has delivered ammunition of the correct type to do this.

If additional ammunition remains available in the unit reserve we allocate a portion of it in a manner that rewards the firing classes. We first determine for each weapon class the actual rate of fire during the previous combat interval based on ammunition expenditure, and make the assumption that a weapon class exhibiting a positive firing rate could have dispensed twenty-five percent more ammunition had it been available, given that the resulting rate of fire would not have exceeded weapon capability. We compute the additional tonnage that a twenty-five percent increase in the rate of fire would require, given the number of weapons currently in the class, and divide the reserve tonnage by fifteen hours to find the hourly tonnage of ammunition that

can be allocated to the high tonnage users. This available tonnage is allocated to each weapon class in proportion to the extra tonnage requirement created by projected usage. Finally we determine for each weapon class the new rate of fire which if continued for fifteen hours would totally exhaust the new tonnage in the classes, and compute the weapon class firepower potential and the unit total firepower potential.

TAGRET SELECTION

Targets selected are retained for the entire combat interval. In the target selection process units are assumed to move in a straight line toward their current objective at a known constant rate for the combat interval. Each unit selects as targets the four opponents projected to be closest during the next combat interval if at the minimum passing distance either the unit or the opponent has a weapon with sufficient firing range.

COMBAT EFFECTIVENESS MULTIPLIER

We assume that a unit at full strength is fully effective and that as personnel casualties mount unit effectiveness falls. We therefore introduce a profile of percent combat effectiveness as a function of percent personnel casualties. At the start of each combat interval the appropriate effectiveness value is determined from this profile. The factor is used both to degrade the firepower output of the unit and also to degrade the unit's ability to move or hold ground.

RANGE EFFECTIVENESS PROFILES

The range effectiveness profile is the device through which the Tarlog model is able to treat attrition at the aggregate unit firepower level while retaining unit descriptions at the weapon class level. The attrition equation used in Tarlog is basically

$$\frac{dL_T}{dt} = -kL_F L_T \qquad (5)$$

where L_F and L_T signify target and firing unit strength, and k is a factor which controls the rate of attrition. The firing unit strength in equation (5) should be effective strength against the target, rather than some independent index of total firepower. The effective firepower that one unit can bring against another is a function of the number and type of weapons in both units, the relative effectiveness of these weapons against each other, and the range between the units at the time of firing. We therefore introduce a set of mission dependent numbers, E_{ijm}, which reflect the probability that weapon i will be in position to acquire weapon j, the probability of acquisition given the opportunity, and the probability of a hit given acquisition. Using the numbers, E_{ijm}, we compute $\bar{E}_{FTR}$, the average effectiveness of F against T at range R.

Let
- R = range between a firing unit F and a target unit T,
- L_{Fi} = firepower potential output rate in the i^{th} class of the firing unit,
- L_{Ti} = firepower potential output rate in the i^{th} class of the target unit,
- L_F = total firepower potential in the firing unit,
- L_T = total firepower potential in the target unit,
- E_{ijm} = relative effectiveness of weapon class i against weapon class j when firing unit has mission m,
- $\bar{R}_i, \underline{R}_i$ = maximum and minimum effective ranges of weapon class i.

The average effectiveness, $\bar{E}_{FTR}$, is defined to be

$$\bar{E}_{FTR} = \sum_i \frac{L_{Fi}}{L_F} \sum_j \frac{L_{Tj}}{L_T} \hat{E}_{ijm}(R) \qquad (6)$$

where

$$\hat{E}_{ijm}(R) = \begin{cases} E_{ijm} & \text{if } \underline{R}_i \le R \le \bar{R}_i \\ 0 & \text{otherwise} \end{cases} .$$

The average effectiveness number, $\bar{E}_{FTR}$, has the property that weapons not in range do not contribute to effectiveness, and that effectiveness depends on the number of units present in a firing class, number of weapons present in a target class, and the relative effectiveness of these weapons. The basic attrition equation then becomes

$$\frac{dL_T}{dt} = - kL_F\bar{E}_{FTR}L_T \tag{7}$$

where $L_F\bar{E}_{FTR}$ is the effective firepower which the firing unit brings to bear against the target unit at range R.

AIR STRIKES

Air strikes using specified air weapons can be directed against specified ground units or against weapon classes within these units. The effect of an air strike is spread uniformly across the combat interval within which the strike is scheduled, and is treated as additional incoming firepower on the target unit. The effectiveness of an air strike against a particular unit is determined through the use of a set of air effectiveness numbers, in a manner similar to that used in the construction of the range effectiveness profile.

COMBAT INTERVAL

The combat interval is subdivided into iteration intervals. At the start of each iteration interval we determine the distance from each unit to each of its targets, determine the range effectiveness factor from the computed range effectiveness profile, allocate fire to each target, and appropriately adjust firepower, location, and possibly mission.

The distance between a unit and each target is used in computing a fire distribution factor which apportions fire to each target in a manner inversely proportional to the square of the distance between unit and target. The firepower output of a unit is reduced due to suppression caused by incoming fire, and further reduced as a function

of the unit's actual movement rate. The remaining firepower is multiplied by the range effectiveness factor, by the fire distribution factor, and by the combat effectiveness multiplier to determine the effective firepower which falls on each target unit. The firepower falling on each target unit is accumulated and used to compute suppression, attrition, and movement degradation to that target during the next iteration interval.

Using the computed movement degradation and the unopposed speed, we calculate the increment in advance or retreat distance, and test for the passage of geographical objectives. If a unit has passed an intermediate objective, its movement is adjusted onto the correct path, and its objective is updated. If an attack unit has reached its final objective it changes to a defensive mission and holds that objective until the end of the iteration interval at which time the game stops. If a delaying unit reaches a defensive holding point it shifts mission to defend that point.

Based on the firepower attrition suffered during each iteration interval, predictions of personnel losses are made. If predicted strength falls below predesignated levels for a unit, that unit will change mission. Attack units can only break, stopping the game. Defensive units will change mission to delay to previously assigned objectives. If delaying units fall below a final break level the game terminates.

During each iteration interval the range between each unit and each target is tested against the range characteristics of each weapon class to provide a precise record of the fraction of the combat interval during which each weapon class was able to fire on the target unit. This is later used to compute ammunition expenditure and individual weapon class losses.

COMBAT INTERVAL ANALYSIS

During the combat interval only unit locations and attrition to aggregate unit firepower are affected. Following the combat interval

we apportion the overall firepower loss into firepower losses in the weapon classes, convert these weapon class firepower losses into numbers of equivalent weapons lost, and then calculate the number of personnel lost. Finally we compute the ammunition expended by each weapon class.

The firepower loss in any individual weapon class is not simply proportional to the original firepower in that class. A weighting factor is derived for each target weapon class which considers the opposing weapon classes that fired at the target unit, the duration that each opposing weapon class fired at the unit, and the effectiveness of each opposing weapon class against each target weapon class, as well as the proportion which the target weapon class firepower bears to the target unit total firepower.

The fraction of weapons lost in the weapon class is calculated to be the same fraction as the weapon class firepower lost during the combat interval.

One of the data entries to the model is the number of personnel lost when a weapon of each class is lost. The number of personnel lost during the combat interval is now computed using the current weapon losses.

Weapon class ammunition expenditure is calculated by first apportioning total firepower output into firepower output by weapon class. This is done by computing a weighting factor to account for the fraction of the combat interval during which each weapon class fired, and the average firepower potential of the class. Using the round weight and the lethal area characteristics of the class, we convert the lethal area expenditure to tonnage expenditure. Following the post-combat interval analysis, preparation begins for the next combat interval. The sequence of operation stops when an attack unit reaches a final objective, any unit's personnel strength falls below a specified level, or the end of game time is reached.

USE OF THE MODEL

In actual operation of the model the restart feature is used extensively. The initial data, units, and conditions are entered

and a combat period of twelve or twenty-four hours is specified. At the completion of this period a restart deck is provided by Tarlog. The progress of the units across the battlefield is plotted, casualty rates are analyzed, and a determination is made as to whether the behavior is reasonable, or whether alternative decisions might have altered the course of the battle at some earlier time. If this is the case, the situation is replayed and terminated at the earlier time, so that a restart deck may be obtained at that time. In any case the restart deck can be changed or added to reflect reinforcement, addition of units, changes in location, objectives, or mission, increases or decreases in resupply rates, or assignment of air strikes. Restart decks may be reproduced to explore several alternatives at interesting branch points, or to parametrize sensitive factors.

IV. CAPABILITIES AND LIMITATIONS

The Tarlog model essentially measures the efficiency with which units convert tons of ammunition to units of lethal area. The attrition suffered by engaged units is proportional to the effective lethal area directed against them. The magnitude of this attrition is ultimately determined by adjusting the factors of the model in a fixed case until some historical or judgmental criterion in satisfied. While this does not allow one to make absolute statements about the outcomes of combat engagements, the results can be used to compare the relative ability of two organizations to inflict casualties on a reference organization.

The Tarlog description of movement and movement degradation is reasonable at an aggregate level. While movement degradation may depend in a direct way on a target unit's weapon mix, convincing descriptions of the mechanism of this dependence do not currently exist. Tarlog will indicate that the organization whose weapons produce the greatest effective lethal area is superior in retarding the advance of an attacker.

With full recognition of the omissions and approximations of Tarlog, we feel it is a useful tool in parametric analysis of force composition.

There are no other combat models currently operating which deal with large combat situations in a less aggregated fashion, or which attempt to use functional relationships between firepower and movement rather than historical tables of degradation values. Tarlog represents a step forward in the attempt to describe the essential elements and interactions of units in large-scale land combat.